Exercise 65

(a) Use the Squeeze Theorem to evaluate $\lim_{x\to\infty} \frac{\sin x}{x}$.

(b) Graph $f(x) = (\sin x)/x$. How many times does the graph cross the asymptote?

Solution

Since $-1 \leq \sin x \leq 1$, use $-1/x$ and $1/x$ for the lower and upper bounds, respectively.

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

Take the limit of all sides as $x \to \infty$.

$$\lim_{x\to\infty} -\frac{1}{x} \leq \lim_{x\to\infty} \frac{\sin x}{x} \leq \lim_{x\to\infty} \frac{1}{x}$$

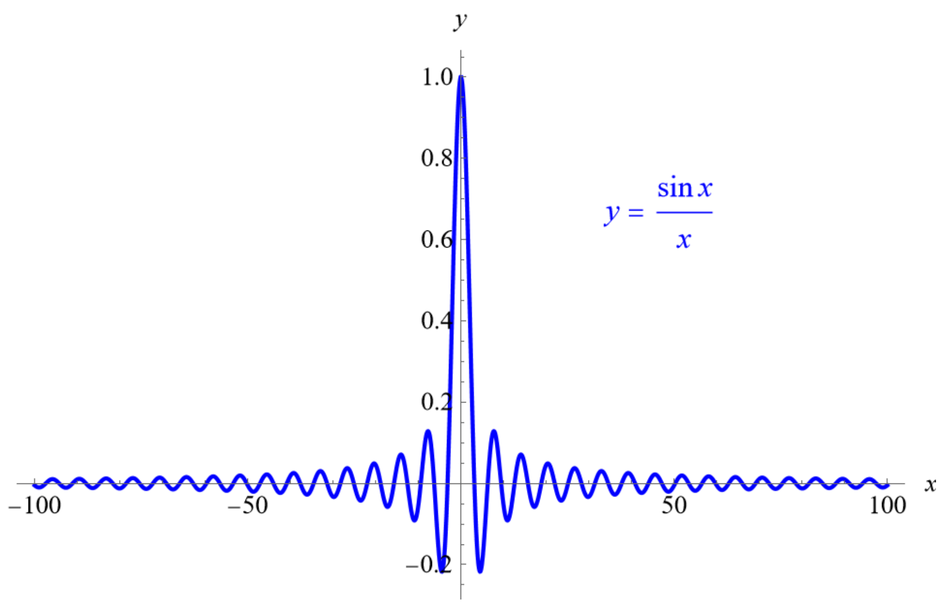
Evaluate the limits.

$$0 \leq \lim_{x\to\infty} \frac{\sin x}{x} \leq 0$$

Therefore, by the Squeeze Theorem,

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0.$$

This is illustrated in the graph of $(\sin x)/x$ below.



The horizontal asymptote is $y = 0$, which is crossed an infinite number of times.